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Millimeter-wave Signature of Strange Matter Stars

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One of the most important questions in the study of compact objects is the nature of pulsars, including whether they consist of neutron matter or strange quark matter (SQM). However, few mechanisms for distinguishing between these two possibilities have been proposed. The purpose of this paper is to show that a strange star (one made of SQM) will have a vibratory mode with an oscillation frequency of approximately 250 GHz (millimeter wave). This mode corresponds to motion of the center of the expected crust of normal matter relative to the center of the strange quark core, without distortion of either. Radiation from currents generated in the crust at the mode frequency would be a SQM signature. We also consider effects of stellar rotation, estimate power emission and signal-to-noise ratio, and discuss briefly possible mechanisms for exciting the mode.

1 Introduction

Witten (1984) pointed out that strange quark matter (SQM) composed roughly of equal numbers of up, down, and strange quarks is more likely to be stable than non-strange quark matter (which would have only up and down quarks but is known not to be stable). This is because conversion to strange quarks (for $m_s < m_N/3$) lowers the Fermi energy. This fact was known to others (see, for example, Bodmer (1971), Friedman and McLerran (1978)). Witten, however, went on to suggest that nuggets of strange quark matter could be produced in phase transitions in the early universe or in supernova explosions and gave a possible scenario for the former, modified versions of which are still under debate; see for example, Cottingham et al. (1994). Witten raised the possibility that such nuggets could solve the cosmological dark matter problem by evading the bound on the cosmological baryon density from the abundance of primordial deuterium. Fahri and Jaffe (1984) considered in some detail the properties of such nuggets as a function of nucleon number. de Rujula and Glashow (1984) considered terrestrial effects, on land, in the sea and in the air, from incident strange quark nuggets.

Alcock, Fahri and Olinto (1986) discussed in depth the conversion of neutron stars to strange stars and the structure and properties of the latter. This work was continued by a number of others and the subject was reviewed thoroughly at a 1991 conference (Madsen and Haensel, 1991). The consensus is that, if SQM is stable, all "neutron stars" should in reality be strange quark stars. In a recent paper, Kettner et al. (1995) have explored the nature of strange quark stars in further detail and computed important properties at non-zero temperatures; this paper takes as its point of departure that paper to which we refer as KWWG.

The object of this paper is to speculate on a possible millimeter wave radio signal that might be a signature of a strange quark star. Because of the fact that there are few observable differences between classical neutron stars and strange quark stars, such a signal could aid in identifying strange quark stars. As far as we are aware, to date only the existence of pulsars with shorter periods than permitted to classical neutron stars (Frieman and Olinto,1989; Glendenning, 1989) and differences in cooling rates (Benvenuto and Vucetich, 1991) have been discussed in detail in the literature. While our proposal is speculative, the impact of detecting strange quark matter would be so great that we believe it important to raise it for discussion and, hopefully, for observational efforts. Our proposal addresses the case generally contemplated by most workers that the strange quark core is surrounded by a crust of normal matter on the order of $10^{-5}M_{\odot}$. We note that, for the case in which the core is almost nude except for an atmosphere of normal matter of much smaller mass, Usov (1997) has recently proposed an X-ray signal.

The plan of the paper is as follows: in the remainder of this section, we summarize the features of the KWWG strange star model relevant to our work. In Section II, we consider the frequency of the vibrational mode in which the crust of hadronic matter vibrates as a single entity, without distortion, with respect to the strange quark core. Radio waves generated from this vibration constitute our proposed signal. In Section III, the effects of the rotation of the strange star are included. In Section IV, we estimate the power that might be radiated if such a mode were excited and the detectability of the resulting signal. Discussion of the results is in Section V.

The essential features, from KWWG, of the core-crust system, as first outlined by Alcock, Fahri and Olinto (1986) are: (i) A strange quark core of roughly $A/3$ each of u-, d-, and s-quarks with a sharp boundary on the order of a fermi where A is the total baryon number. The boundary is sharp because the core is bound by the strong force, not gravity. (ii) An electron gas extending a few hundred fermis beyond the core. The electron abundance, which is on the order of $10^{-4}A$, would be zero if the strange quark mass were essentially zero, as is true for up and down quarks, rather than the estimated $100 - 300$ MeV/c^2. The electron gas beyond the core is held by, and accompanied by, a strong positive electric field resulting from the net positive charge on the sharp core; it is the gradient of a megavolt range potential. (iii) A hadron crust. Non-strange matter attracted gravitationally by the core has its electrons repelled by the Pauli pressure of the electron gas and its ions repelled by the electric field. Neutrons suffer neither of these repulsions. A crust can therefore accumulate until it becomes energetically favorable for neutrons to leave nuclei at the base of the crust and to "drip" into the core.

The mass of the crust is bounded by $M_C \leq 10^{-5} M_\odot$ for SQM star mass, $M_Q \simeq M_\odot$. KWWG note that the electrostatic potential inside the strange quark core is $eV(r) = \mu_e(r)$, where μ_e is the chemical potential for which they solve numerically along with the quark chemical potential. They choose specific values for the mass of the strange quark (150 MeV) and the MIT bag constant (50 MeV/fm), which parametrizes quark confinement in QCD. They find, using local charge neutrality, $eV(r)$ near the surface ($r = R$) of the core, about 18.5 MeV for zero temperature, with quadratic corrections for finite temperature bringing $eV(r)$ near the surface down by about 0.5 MeV at T=50 MeV. Using global charge neutrality just at the surface, $eV(r)$, for zero temperature, falls to 3/4 its (nearby) interior value; it falls to about half the interior value for T=50 MeV. KWWG solve Poisson's equation in the gap between the core and the hadron crust. There are two constants of integration. They can be taken as the potentials at the outer edge of the core ($r = R$) and the inner edge of the crust ($r = R_C$). The scale length over which the potential falls only depends on the first and is of the order of the few hundred fermis. The width of the gap is given by the value of r at which the potential has fallen to the second. More precisely, they show

$$eV(r) = \frac{C}{r - R + r_0}, \qquad R < r < R_C \tag{1}$$

with $r_0 \equiv \frac{C}{eV(R)}$ and

$$C = (3\pi/2)^{1/2}/e \;=\; 5 \times 10^3 \mathrm{MeV-fm} \;=\; 8.5 \times 10^{-16} \mathrm{erg-cm}. \tag{2}$$

2 Vibrations

Figure 1 shows the centers of the core and crust displaced along the polar axis by $\xi < \Delta_G$, where Δ_G is the width of the gap. We need to compute the restoring force. First, we note that for $\xi = 0$, the electrostatic repulsion and gravitational attraction balance. The electrostatic repulsion pressure is given by

$$P_{el} = \frac{Ze\eta_A C}{(\Delta_G + r_0)^2} \tag{3}$$

where η_A is the number of ions per unit area at the base of the crust and Ze is their average charge. The gravitational attraction pressure is

$$P_G = \frac{GM_Q M_C}{4\pi R^4} \tag{4}$$

For $\xi << (\Delta_G + r_0)$, we have

$$r^2 = R_C^2 \sin^2\phi + (R_C \cos\phi - \xi)^2; \qquad r \simeq R_C - \xi\cos\phi \tag{5}$$

and so

$$\begin{aligned} F_z(\xi) &= \xi\, Ze\eta_A C \int_0^\pi 2\pi\, \sin\phi\, d\phi\, \frac{d^2}{dz^2}[r - R_Q + r_0]\Big|_{z=0} \\ &= -\frac{8\pi}{3}\, \frac{\xi\, Ze\eta_A C}{(\Delta_G + r_0)^3} \\ &= -2/3\,\xi\, \frac{GM_Q M_C}{R^2}\, (\Delta_G + r_0)^{-1} \quad . \end{aligned} \tag{6}$$

The result is

$$\omega^2 = \frac{2GM_Q}{3R^2(\Delta_G + r_0)} \tag{7}$$

For a strange star at zero temperature with a maximal crust, we have $\Delta_G \sim 200$ fm and $r_0 \sim 300$ fm. From Equation (7), we can see that $\nu_0 \simeq 2.5 \times 10^{11}$ Hz. and $\lambda = 1.2$ mm. If the temperature rises to 50 MeV, $V(r)$ falls by 25% according to KWWG, so we then have $\nu_{50} = 2.6 \times 10^{11}$ Hz and $\lambda_{50} = 1.4$ mm.

We can also ask how ν varies if M_C is reduced from its maximum value when $\Delta_G \sim 200$ fm and $\rho \simeq 4.3 \times 10^{11}\ \mathrm{g\,cm^{-3}}$. The equality of (3) and (4) gives

$$(\Delta_G + r_0)^2 = R^4\, \beta^2 \tag{8}$$

where

$$\beta = \left(\frac{4\pi ZeC\eta_A}{GM_Q M_C}\right)^{1/2} \tag{9}$$

and

$$\eta_A = \left(\frac{\rho}{Am_p}\right)^{2/3} \tag{10}$$

In Equation (9) the quantity A is the average atomic number and m_p is the mass of the proton. The dependence of ρ on M_C can be found by relating ρ to the pressure at the base of the crust in Equation (4) by means of the equation of state. Using the results of Harrison and Wheeler (Harrison et al., 1958 and 1965) and of Baym, Pethick and Sutherland (1971)

as discussed in Shapiro and Teukolsky(1983), we have, for $10^8 < \rho < 4.3 \times 10^{11} = \rho_c$, by interpolating from the numerical results,

$$\rho \simeq \rho_c (P/P_c)^{5/6} \qquad \text{and} \qquad P_c = 10^{29.5} \text{dynes/cm}^2 \tag{11}$$

Inserting into Equation (8), we see that $(\Delta_G + r_0) \propto M_C^{-2/9}$ and hence $\lambda \propto M_C^{-1/9}$. Thus, a decrease in M_C by a factor of 100 increases the wavelength by only a factor of 1.7.

We thus find that a low-temperature crust of maximal mass should exhibit a signal at about 1.2 mm; the wavelength increases with increasing temperature and decreasing crust mass. A 50 MeV crust with one percent of the maximal mass would have a wavelength of about 2.4 mm. We note that the lower bound on the wavelength (1.2 mm)) is well above the region in which the atmosphere becomes opaque.

The calculation above ignores the work done against the Pauli pressure of the electrons. We estimate that effect using (see KWWG) for the Pauli pressure

$$P(r) = \frac{\mu^4}{12\pi^2} \simeq \frac{(4eV(r))^4}{12\pi^2}. \tag{12}$$

Taking the gradient, we find $16\pi R^2 P(r) F_G^{-1} < 0.05$ where $F_G \simeq 4\pi R^2 P_G$. Thus, electron Pauli pressure is a small effect. Finally, we note that no new modes are introduced by angular motion of the crust center of mass about the core center of mass because of the spherical symmetry. However, if such motion were induced, the centrifugal term $L^2/(M_C R^3)$ would effectively weaken P_G in Equation (4). It would not affect ω^2 because, as with the gravitational force, it changes on a scale of order R, not one of order r_0 as with the electric force. It could, however, modulate the angular distribution of radiation from the system.

In summary, for strange quark star crusts of mass M_C in the range $10^{-7} M_Q < M_C < 10^{-5} M_Q$ with temperatures below about 50 MeV, the crust-core system has a normal mode corresponding to a wavelength λ roughly in the region 1.2 to 2.4 mm.

3 Rotational Effects

Any real strange star will be rotating. One expects rotational periods ranging from milliseconds to a few seconds. This should have two effects on the millimeter wave signature. The first is simply Doppler broadening–a 10 km diameter star rotating at 1 Hz would make the signal bandwidth $25 \sin\theta$ Mhz, where θ is the angle between the rotation axis and the observer, at an observing frequency of 250 GHz. The bandwidth clearly scales proportional to the frequency. The second effect is caused by the fact that the rotation will cause the star to become oblate, leading to two normal modes of oscillation and thus splitting the signal. This is similar to the giant resonance mode in nuclei, in which the mode is split into two modes in nuclei with spin. In this section, we calculate the frequency splitting.

For a non-rotating strange star, the zero-temperature equation of state is $P = \frac{1}{3}(\rho - 4B)$, where B is the bag constant. This equation is inserted into the Oppenheimer-Volkov (OV) equation of hydrostatic equilibrium

$$\frac{d\rho}{dr} = -\frac{(\rho + P)(m + 4\pi r^3 P)}{r(r - 2m)} \tag{13}$$

where we use units $c = G = 1$ and m is the mass inside the radius r (and $2m$ is its Schwarzschild radius). The result of integrating the OV equation (Alcock, Farhi, Olinto, 1986) gives the structure of the star. If it is rotating, there will be an additional centrifugal pressure term added to P in the OV equation, $P \to P - \frac{1}{2}\rho\omega^2 r^2 \sin^2\theta$. Integrating the OV equation again with this term along the polar and equatorial directions gives the polar and equatorial radii, shown in Table 1, and the resulting eccentricity, as a function of the angular velocity of the star, Ω. These results were obtained in the approximation of small eccentricity. At higher order, considerations of the mass distribution within the star, relativistic corrections, the difference between the shape of the inner edge of the crust and that of the core, etc. must be included.

The strange star is now an oblate spheroid. One will find two normal modes, corresponding to vibrations in the polar and equatorial directions. We find that (again, to leading order in the eccentricity)

$$\Delta\omega = \left(\frac{1}{R^2_{polar}} - \frac{1}{R^2_{equat.}}\right)\frac{2GM}{3\beta} \tag{14}$$

This frequency splitting, $\Delta\omega/\omega$, is also given in Table 1. We can see that for strange stars with periods of the order of seconds, the frequency splitting is negligible relative to the Doppler broadening, whereas for strange stars with periods of the order of milliseconds, the two are of the same order of magnitude. For such stars, this splitting would be a significant indicator of a strange star origin of a narrow line, millimeter wave signal from a pulsar.

It should also be noted that if the pulsar is in a binary system, the tidal distortion would also cause a splitting of the mode, and the above equation would apply. (If the pulsar were rapidly rotating as well, the deformation would be quadrupolar.) The calculation of the tidal force (ignoring rotation) is straightforward, and it is easy to show that the results are identical with ω^2 replaced by $\omega_0^2/4$, where ω_0 is the frequency of revolution. Since the frequency of revolution, for all realistic pulsars, is much less than the frequency of rotation, the effects of tidal distortion will be negligible.

4 Radiation and Detectability

The energy stored in the vibrational mode of Section 2 is given by

$$E \sim \frac{1}{2} M_C \xi^2 \omega^2. \tag{15}$$

Whereas the frequency of the mode is nearly independent of crust mass (varying roughly as $M_C^{1/9}$), the energy stored goes roughly as the 11/9 power. For $M_C \sim 10^{-5} M_\odot$ and $\xi \sim 200$ fm, E is of the order of 10^{31} ergs. We estimate the radiation rate in a simple model. When the crust center of mass is displaced downward, relative to the core center of mass by ξ, as shown in Figure 1, the electrostatic potential at the "top" rises by $-\xi\frac{dV}{dx} > 0$ and the potential at the "bottom" falls by the same amount. However, the crust is an equipotential, made from a material of very high conductivity, although likely not a superconductor, so charge must flow to cancel this change. Consider the "flat star approximation" in which the crust consists of two parallel planes, each of radius R and with separation R. To maintain

the equipotential, a sheet of charge density σ must flow with current $I = \omega Q$,

$$\Delta V = 4\pi\sigma R = 2\xi \frac{dV}{dz}, \tag{16}$$

and

$$Q = \pi R^2 \sigma = R\frac{\Delta V}{4}, \tag{17}$$

giving radiation power on the order of

$$P = \frac{dE}{dt} = \frac{I^2}{2c} \simeq (\omega \xi R V')^2 \tag{18}$$

This expression will only be valid for temperature not too much smaller than the 5 MeV Fermi momentum of the electrons in the crust. For very small T, we would expect the radiation rate to fall like $(T/p_F)^3$ as Pauli blocking makes electrons (and holes) unable to radiate at frequency ν. For $\xi V' \sim 10$ MeV/e, Equation (18) gives $P \sim (10\, eV/e)^2 (R\omega)^2/c \sim 10^{34}$ erg/s, which is a very large signal. This rate is reduced to the extent that radiation from electrons not at the surface will either not occur or will be absorbed and simply heat the crust. One rough estimate of this reduction would be to assume that the charge is spread evenly throughout the crust and hence reduce the intensity of the radiation by $\lambda/\Delta_C \sim 10^{-5}$, where $\Delta_C \sim 100m$ is the crust thickness. However the crust conductivity is so high, $\sigma \sim 10^{25} s^{-1}$, (Pethick and Sahrling, 1995) that the skin depth (Jackson,1975) is far less than the wave length. For a conservative estimate of this effect we take the geometric mean between λ and λ/Δ_C giving $P_{rad} \sim 10^{-3} P$. Thus, a rough power estimate might be 10^{31} erg/s, for maximal excitation of the mode and for a crust of maximum thickness. The power, but not the decay time, would scale with the energy in the mode. Roughly, the decay time would scale with the thickness of the crust and the power radiated would scale inversely.

At a distance D in kpc from the pulsar, the flux density of radiation emitted at the rate of P Watts spread over a frequency f (Hz) is

$$S = 8.3 \times 10^{-15} \frac{P}{f D^2} \text{ Jy} \tag{19}$$

where 1 Jy is 10^{26} W/m^2/Hz. A 10 meter sub-millimeter telescope operating at a frequency of 250 GHz in good conditions near zenith has an rms noise

$$\Delta S = \frac{7000}{(ft)^{1/2}} \text{Jy} \tag{20}$$

where the integration time, t, is in seconds. The signal to noise ratio for a continuous signal is

$$\frac{S}{\Delta S} = 1.2 \times 10^{-18}\ P \left(\frac{t}{f}\right)^{1/2} \frac{1}{D^2} \tag{21}$$

For a pulsar of period 1 second and a consequential Doppler broadening of ~ 25 MHz (at an observing frequency of 250 GHz) emitting a 10^{24} W signal, a 15 second integration time yields a signal-to-noise ratio $\sim 10^3 D^{-2}$. A detectable signal (5σ) could be achieved for pulsars as

far away as 15 kpc (there are more than 100 with periods in excess of 1 second). For a pulsar as close as 1 kpc (there are about ten with periods longer than 1 second) a 10^{29} erg s^{-1} signal could be detected. The strength of the signal is not the problem for detection; the problem is not knowing the frequency. Receivers in the 250 GHz range have instantaneous bandwidths of a few GHz which could be searched in a few minutes with a 1 GHz spectrometer. Since retuning the receiver to the next band would take perhaps a quarter hour, most of the time for the search would be spent in retuning the receivers, not in the observing.

5 Discussion

One of the most important questions in the study of compact objects is the nature of pulsars, including whether pulsars consist of neutron matter or strange quark matter. In this paper, we have identified an observable radio signal that would be characteristic of the latter possibility. A spectral line originating from the pulsar could easily be distinguished from one originating in the interstellar medium because the pulsar line will be detected only when the telescope is pointed at the pulsar. We have, however, left important questions unanswered. How is the vibrational mode excited? A transient signal, such as that due to a starquake or cometary impact, would die off quickly, on the order of milliseconds, and would thus be unobservable. Does a mechanism for continuous excitation exist? The power of a detectable signal is only on the order of 10^{-6} of pulsar energy losses. Thus it would seem reasonable, in view of the many consequences, to make observations – independent of theoretical considerations, in order to see if such a small fraction of the energy loss does, in fact, go into this mode. We may speculate, however, that one direction from which such a mechanism could come might be that of the interaction between superfluid vortices and (type 2) superconducting magnetic flux tubes. (We know that the magnetic field for a strange quark star must pass through the core because the crust is so thin.) It is the outward migration of the superfluid vortices that is responsible for pulsar spin-down, while Ruderman (1996) has conjectured that the latter could be effectively pinned by the interaction with the non-superconducting electron fluid. Some effective resonant coupling between those two systems, on the one hand, and the vibrational mode, on the other, could result in continuous excitation. For example, the coupling could be associated with discontinuous passage of a vortex through a pinned flux tube.

Even if the mode is excited, we would expect some fraction of the energy stored in the mode to go into heating the crust, and some fraction to go into radiation. Our crude estimate that $P_{rad}/P \sim 10^{-3}$ of the energy goes into radiation needs to be improved. Another question concerns the scale over which the coherence implicit in our calculations is maintained. If such coherence is only maintained over some fraction of the size of the star, then the power radiated would be reduced by that fraction.

The signal predicted in this paper is a speculative one; however the dearth of distinctive signatures for strange quark stars, in our view, makes the search for such a signal worthwhile.

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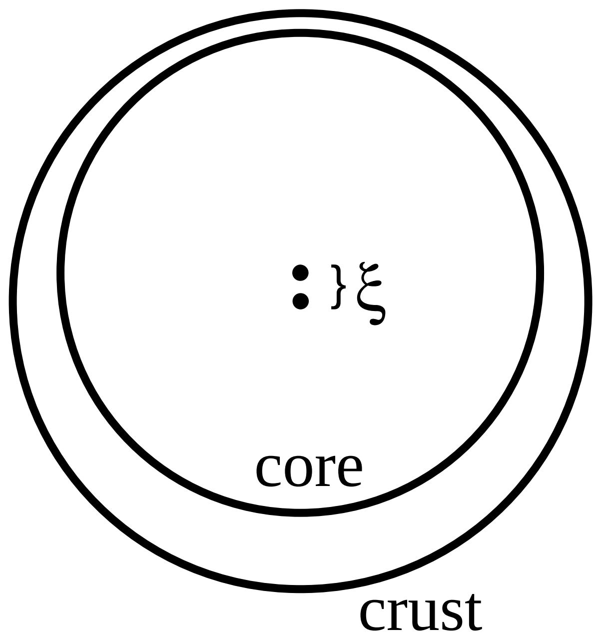
TABLE 1

Rotational Frequency(Hz)	Doppler Broadening(GHz)	eccentricity	Mode Splitting(GHz)
10	.25	6×10^{-6}	.0015
100	2.5	5.5×10^{-4}	0.13
1000	25	.058	15

Table 1 For strange stars rotating with angular velocities from 1-1000 Hz, we give the expected Doppler broadening of the 250 GHz signal, the eccentricity of the star and the resulting splitting of the signal. We give results for a strange star central density of $5.5B$, where B is the bag constant; the results of the mode splitting will vary by roughly a factor of two for the expected range of central densities. The Doppler broadening results assume the observer is in the equatorial plane; if this isn't the case, the results should be multiplied by $\sin\theta$.

Figure Caption

Figure 1. The figure shows the centers of the core and crust being displaced. The resulting oscillation has a frequency of approximately 250 GHz, leading to the millimeter wave signal discussed in the paper.



$\}\xi$
core
crust